Modern English School
Cairo

American Section Mathematics Department

Summer work for Pre-calculus students

Please complete the following questions showing full work. Submit to your Precalculus teacher on the first day of term.

I. Quadratics Equations

Solve each of the following equations using factoring.

a. $x^2 - 36 = 0$	b. $7x^2 - 14x = 0$
c. $x^3 - 6x^2 - 7x = 0$	d. $6x^2 + 7x - 3 = 0$
e. $3x^2 + 3x - 36 = 0$	f. $32x^2 - 2 = 0$
g. $x^3 - 2x^2 - 9x + 18 = 0$	h. $x^3 - 3x^2 + 6x - 18 = 0$

Factor each polynomial COMPLETELY.

Sum of Two Cubes: $\boxed{a^3+b^3=(a+b)(a^2-ab+b^2)}$

Difference of Two Cubes: $\boxed{a^3-b^3=(a-b)(a^2+ab+b^2)}$

a. x^3+27

b. $8x^3-125$

c. x^4+5x^2-14

d. $2x^5-18x^3+40x$

Solve each of the following equations using the Quadratic Formula.

Quadratic Formula: $\boxed{x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}}$

a. $4x^2+6x+1=0$

b. $x^2+2x+2=0$

c. $2x^2+3x-5=0$

d. $3x^2-2x-7=0$

II. Powers, Roots, and Radicals

Rewrite the expression with positive exponents. Evaluate where possible.

a. $(-3)^{-4}$

b. $\dfrac{4}{x^0+7}$

c. $3x^3(2x)^2$

d. $\dfrac{8a^4b^6}{2(a^5b)^2}$

e. $4(x^{-3}y^4)(-3xy^2)^2$

f. $\dfrac{20(a^{-4}b^{-2})}{8(a^{-2}b^4)^{-2}}$

Solve the radical or rational exponent equation.

a. $x^{\frac{1}{5}} = 2$

b. $2\sqrt{3x-1}+3=11$

c. $4x^2 = 64$

d. $2(x-2)^{\frac{1}{4}} - 3 = 159$

e. $\sqrt{2x+4} = \sqrt{x+2}$

f. $\sqrt[3]{x} - 6 = -2$

III. Simplifying Rational Expressions

Simplify the Rational Expression using Multiplication or Division.

a. $\dfrac{x^2+4x-12}{x^2\left(x^2+9x+18\right)}\cdot 6x^2$

b. $\dfrac{3x^2-12}{5x-10}\cdot\dfrac{1}{2x+4}$

c. $\dfrac{x^2-4}{x^2+4}\cdot\dfrac{x+2}{x-2}$

d. $\dfrac{5x^2-20}{25x^2}\cdot\dfrac{x}{x-2}$

e. $x^2+x-30\cdot\dfrac{x}{x^2+6x}$

f. $\dfrac{12x^2y^3z}{6x^3y^2z^2}$

g. $\dfrac{x^3+3x^2}{2x}\div\dfrac{x^2+5x+6}{5x^3}$

h. $\dfrac{x^2+x-20}{x+1}\div\dfrac{11x+55}{x+1}$

i. $\dfrac{x^2+5x+6}{x+3}\div\dfrac{x^2-4}{x+1}$

j. $\dfrac{x^2+6x-7}{3x^2}\div\dfrac{x+7}{6x}$

Simplify the Rational Expression using Addition or Subtraction. (LCD = ?)

a. $\frac{4}{3x^2}+\frac{2}{5x}$

b. $\frac{3}{2x-2}+\frac{x+1}{4}$

c. $\frac{4}{3x^3}+\frac{x}{6x^3+3x^2}$

d. $\frac{5x-1}{x^2+2x-8}-\frac{6}{x+4}$

e. $\frac{x+1}{x^2+6x+9}-\frac{1}{x^2-9}$

IV. Solving Rational Equations

Solve each rational equation.

a. $\frac{3}{x+4}=\frac{9}{x-2}$

b. $\frac{4x}{x-1}=\frac{x}{x^2-1}$

c. $\frac{3}{x^2-4}=\frac{2}{x+2}+\frac{x}{x-2}$

d. $\frac{3x-2}{x-2}=\frac{6}{x^2-4}+1$

e. $\frac{x}{x+2}=\frac{3x+1}{x-1}+\frac{4}{x^2+x-2}$

V. Function Operations

Perform the indicated operation with the functions given.

Let $f(x)=x^2-3x+4$, $g(x)=5x+2$, and $h(x)=6x$.

a. $(f+g)(x)=$	b. $(f-h)(x)=$	c. $(g\cdot h)(x)=$
d. $(f+h)(-2)=$	e. $(h-g)(3)=$	f. $(g\cdot f)(0)=$
g. $(f\circ g)(x)=$	h. $(f\circ h)(x)=$	i. $(g\circ f)(1)=$
j. $(f\circ h)(-7)=$	k. $g(h(f(x)))=$	l. $f(g(h(-1)))=$

VI. Inverses

Find the inverse of each function.

a. $f(x)=2x+5$

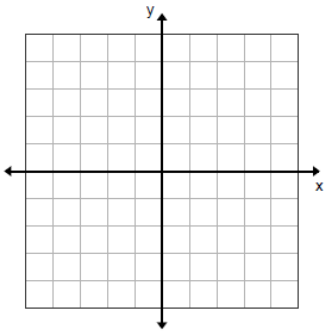
b. $f(x)=\sqrt[3]{2x+4}$

c. $f(x)=5-\frac{5}{2}x$

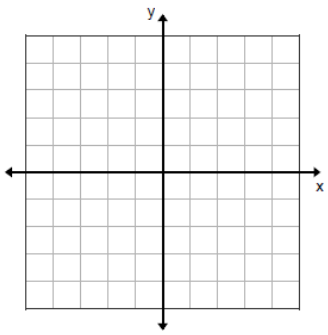
d. $f(x)=\frac{x-2}{4}$

Verify that the two functions are inverses of each other using composite functions. Then, verify (a) and (b) by graphing.

a. $f(x)=x+7,\ g(x)=x-7$



b. $f(x)=\frac{1}{2}x+1,\ g(x)=2x-2$



c. $f(x)=\frac{1}{3}x^2,\ g(x)=\sqrt{3x}$

d. $f(x)=\frac{x^5+2}{7},\ g(x)=\sqrt[5]{7x-2}$

The graph of the inverse function is the reflection of the original function over what line?

VII. Exponential & Logarithmic Functions

Solve each equation.

$y = \log_b x$ if and only if $x = b^y$.

Think of $y = \log_b x$ as the answer to: "To what power must *b* be raised to obtain *x*?"

a. $\left(\frac{1}{3}\right)^x = 27$	b. $5^{3x} = 25^{x-1}$	c. $4^x = 0.25$
d. $10^x = 15$	e. $e^{3x} = 24$	f. $\ln 3x = -0.5003$
g. $\log_x 64 = \frac{1}{2}$	h. $\log_3 x = 5$	i. $\log_4 256 = x$
j. $\log_7(2x+5) = \log_7(x-3)$	k. $\log_2(2x^2) = 5$	l. $\log x = 2.096910013$
m. $256e^{2x} = 1400$	n. $75 = 21(1.05)^t$	o. $10^{x^2+3x-7} = 1{,}000$

Write the logs in condensed form.	Write the logs in expanded form.
a. $2\log x - x\log y$	b. $\log x^2 y^3 z^4$
c. $\log x + 2\log y$	d. $\log(x^2+1)z$
e. $\log x + \frac{1}{2}\log y - 2\log z$	f. $\log\frac{x^2}{z^6}$
g. $\log x + \log y + \log z - 2\log w$	h. $\log x^2 y$

Use the equation given and the properties of logs to solve the problems below:

(1) $A = P\left(1+\frac{r}{n}\right)^{nt}$

(2) $A = Pe^{rt}$

where:

- P = original amount deposited or the initial investment
- r = the interest rate expressed as a decimal (5% → 0.05)
- n = the number of times a year the interest is paid ("quarterly" → means n = 4)
- t = the number of years the investment spans

a. Find the value of a $1,000 investment at 6% interest after 10 years compounded:

(a) annually

(b) quarterly

(c) monthly

(d) continuously

b. If you invest $30,000 at 4.76% interest paid quarterly, how long would it take you to double your money? Round your answer to the nearest hundredth.

c. Suppose $2,000 is invested in a 3-year certificate of deposit (CD) that earns 6% interest, compounded continuously. How much will the investment be worth after 3 years?

d. You invest $200 at 12.25% earning continuous interest. How many years does it take for your money to increase 5 times its original value? Round your answer to the nearest tenth.

Simplify:

1. $-\sqrt[5]{-32}$
2. $\sqrt[4]{16x^8y^4}$
3. $\sqrt[3]{108} - 2\sqrt{75} + \sqrt{147}$
4. $\sqrt[3]{12x^3}\sqrt[3]{2x^4}$
5. $(2+\sqrt{5})(1-\sqrt{5})$
6. $\sqrt[3]{x}(\sqrt[3]{3x^2} + \sqrt[3]{12x})$
7. $\frac{\sqrt{60a^8}}{\sqrt{72a^5}}$
8. $\frac{1}{2-\sqrt{3}}$

Write the rational exponents in radical form:

9. $4^{\frac{2}{3}}$
10. $y^{\frac{1}{4}}$

Solve:

11. $\sqrt{5x+4} = 12$
12. $\sqrt{4x+1} - \sqrt{x-2} = 3$

Simplify:

13. $\sqrt{-3} \cdot 5i$

14. $(8 - 5i) - (-1 + 3i)$

Multiply:

15. $(8 - 5i)(8 + 5i)$

16. $\left(2 - i\sqrt{3}\right)(2 + i\sqrt{3})$

Divide:

17. $\frac{2}{4i}$

18. $\frac{(7+2i)}{4+5i}$

Solve by factoring:

21. $b^2 = 4b + 21$

22. $x^2 = 8x$

Solve by the square root method:

23. $q^2 - 169 = 0$

24. $(2x - 4)^2 = -64$

Solve by completing the square:

25. $x^2 + 12x + 18 = 0$

26. $2x^2 - 7x = 15$

Solve using the quadratic equation:

27. $x^2 - 3x + 5 = 0$

28. $4x^2 - 2x + 5 = 0$

Find the discriminant and determine the number of solutions:

29. $x^2 + 3x + 5 = 0$

30. $10 - 2x^2 - 6x = 0$

Test the following equations for symmetry with respect to the x and y axis:

31. $4y = 3x^2 - 1$

32. $2x^2 - 3y^2 = 5$

33. $x^3 + 3y^3 = 10$

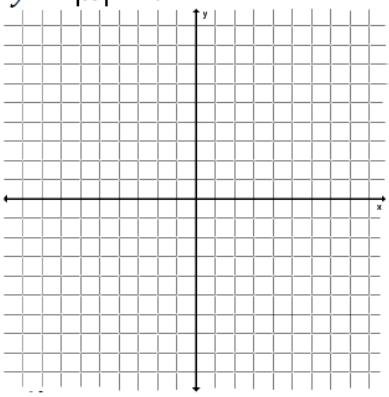
Determine whether each function is even, odd, or neither:

34. $f(x) = 5x$

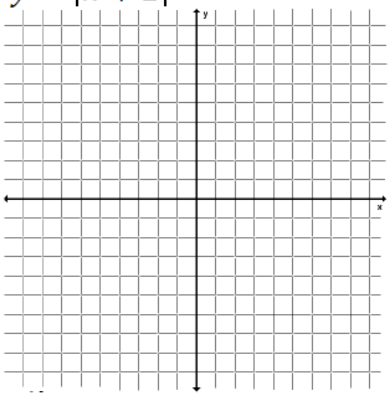
35. $f(x) = x^2 + 3$

Given the graph $y = |x|$ graph the following:

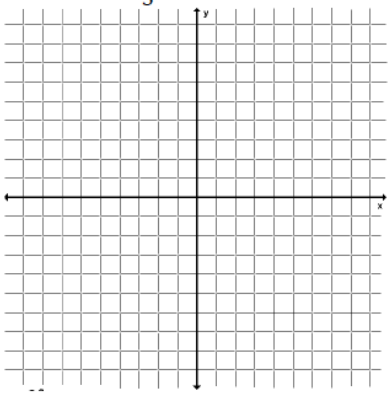
36. $y = |x| - 7$



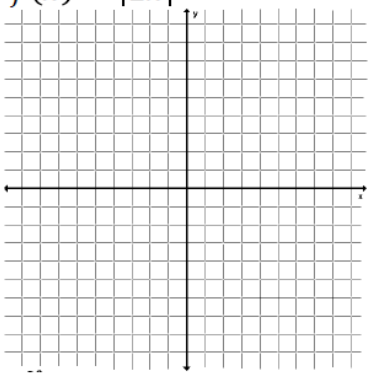
37. $y = |x + 2|$



38. $f(x) = -\frac{1}{3}|x|$

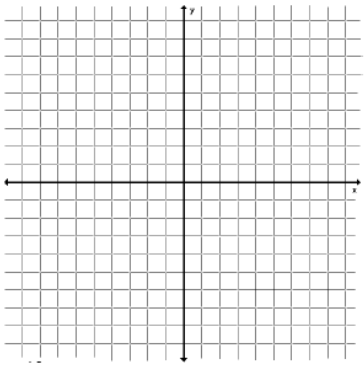


39. $f(x) = |2x|$

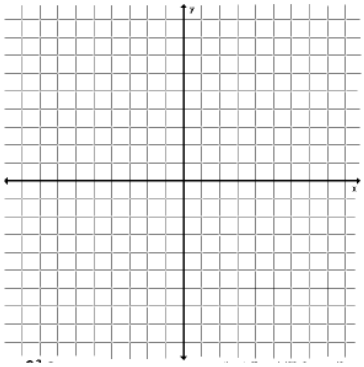


Graph the quadratic function and determine the vertex, line of symmetry, and minimum/maximum value:

40. $f(x) = -2(x+6)^2 + 3$



41. $f(x) = \frac{1}{4}(x-4)^2 + 2$



Rewrite the quadratic function in standard form by completing the square:

42. $f(x) = x^2 - 3x + 5$

43. $f(x) = -4x^2 + 2x + 3$